

# Heat and Thermodynamics

AIEEE Syllabus

Heat, temperature, thermal expansion; specific heat, calorimetry; change of state, latent heat; Thermal equilibrium, Zeroth law of thermodynamics; Heat, work and internal energy. First law of thermodynamics; Carnot engine and its efficiency

# 9

## CHAPTER

### THIS CHAPTER COVERS :

- Thermal Expansion
- Coefficient of Apparent Expansion of a Liquid
- Specific Heat
- Molar Specific Heat
- Latent Heat
- Internal Energy
- First Law of Thermodynamic
- Different Thermodynamic Processes
- Work Done by System in Different Processes
- Indicator Diagram
- Slope of Isothermal and Adiabatic Curves
- Carnot Engine and Heat Pump
- Efficiency of Carnot Cycle and Carnot Theorem

### HEAT

When a hot body is kept in contact with a cold body, there is a transfer of energy from hot body to cold body. The energy transferred is called heat.

**ZEROTH LAW OF THERMODYNAMICS** : If a body *A* is separately in thermal equilibrium with body *B* and body *C* then *B* and *C* are also in the thermal equilibrium.

“Two bodies which are in thermal equilibrium are said to have equal temperatures”.

### Thermal Expansion

When the temperature of a body increases, its size increases.

(1) Coefficient of linear expansion is given by

$$\alpha = \frac{\Delta L}{L\Delta T}$$

$$L_{\theta} = L_0 (1 + \alpha\theta)$$

(2) Coefficient of superficial expansion is given by

$$\beta = \frac{\Delta A}{A\Delta T}$$

$$A_{\theta} = A_0 (1 + \beta\theta)$$

(3) Coefficient of cubical expansion is given by

$$\gamma = \frac{\Delta V}{V\Delta T} \quad \text{or} \quad V_{\theta} = V_0 (1 + \gamma\theta)$$

$$\frac{m}{\rho_{\theta}} = \frac{m}{\rho_0} (1 + \gamma\theta) \Rightarrow \rho_{\theta} \approx \rho_0 (1 - \gamma\theta)$$

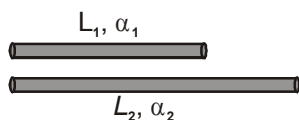
An Isotropic body expands equally in all directions and we can obtain the following relations

$$\gamma = 3\alpha, \beta = 2\alpha \quad \text{or} \quad \boxed{\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}}$$

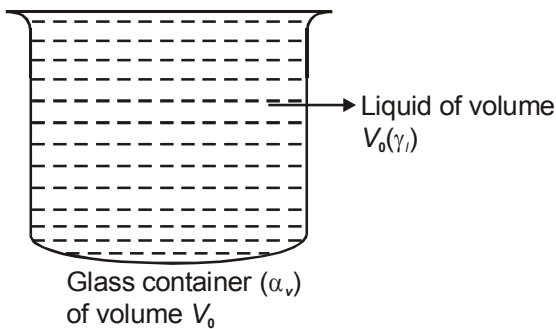
**Applications**

- (1) If  $\alpha_x, \alpha_y, \alpha_z$  are coefficient of linear expansion along x-axis, y-axis and z-axis, (i.e., for anisotropic body) then  $\gamma = \alpha_x + \alpha_y + \alpha_z$ .
- (2) For water  $\gamma$  is negative between 0°C and 4°C
- (3) Density of water is maximum at 4°C. Therefore water at the bottom of lake in winter is warmer than that at the surface.
- (4) Two rods of length  $L_1$  and  $L_2$  are kept side by side. If with increase in temperature, the difference in their lengths does not change i.e.  $L'_2 - L'_1 = L_2 - L_1$  where  $L'_2 = L_1(1 + \alpha_1\Delta t)$  and  $L'_2 = L_2(1 + \alpha_2\Delta t)$

then  $L_1\alpha_1 = L_2\alpha_2$  or  $\frac{L_1}{L_1 - L_2} = \frac{\alpha_2}{\alpha_2 - \alpha_1}$ .



- (5) A vessel is completely filled with a liquid  
 $\alpha_v$  = coefficient of linear expansion of vessel  
 $\gamma_l$  = coefficient of cubical expansion of liquid  
 With increase in temperature, the volume of liquid flown out is given by



$\Delta V = V_0[1 + (\gamma_l - \gamma_v)\Delta\theta] = V_0[1 + (V_l - 3\alpha_v)\Delta\theta]$   
 $\Rightarrow \Delta V = V_0[1 + \gamma_a\Delta\theta]$ , where

$\gamma_a$  = coefficient of apparent expansion =  $\gamma_l - \gamma_v = \gamma_l - 3\alpha_v$

- (6) Variation of moment of inertia with temperature

$I' = I(1 + \beta\Delta\theta)$

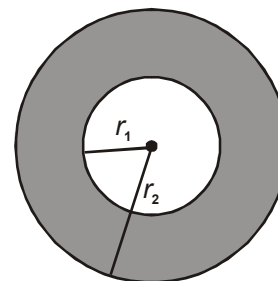
Also,  $\frac{\Delta I}{I} = \beta\Delta\theta = -\frac{\Delta\omega}{\omega}$  (When Angular momentum  $L = I\omega = \text{Constant}$ )

- (7) As an annular disc is heated, all the dimensions (including cavity) increase.

Thus, we have

$r'_1 = r_1 (1 + \alpha\Delta\theta)$

$r'_2 = r_2 (1 + \alpha\Delta\theta)$



**Annular disc**

In other words, you can say that the thermal expansion is photographic i.e. as when a photograph is enlarged, all the dimensions of the photograph increase.

- (8) Coefficient of volume expansion of an ideal gas at constant pressure is given by

$\gamma = \frac{1}{T}$

- (9) A meter scale calibrated at  $T_1$ °C is used for measurement at  $T_2$ °C. Let  $\alpha$  be the coefficient of linear expansion for scale, and gives a reading  $L$ , then error in the reading is  $\Delta L = \alpha L(T_2 - T_1)$ .

**CALORIMETRY**

**Specific heat capacity :**  $c = \frac{\Delta Q}{m\Delta T}$  (cal/g/°C)

**Molar heat capacity :**  $C = \frac{\Delta Q}{n\Delta T}$  (cal/mol/°C)

**Note :** Molar heat capacity = (Mol. mass) × Specific heat capacity

**Latent heat**

(1) of fusion  $L_f = \frac{Q}{m}$

(2) of vaporisation  $L_v = \frac{Q}{m}$

**Water**

Specific heat  $C = 1$  cal/gm/°C = 4.2 J/gm/°C = 4200 J/kg/°C

$L_f = 80$  cal/gm = 336 J/gm

$L_v = 540$  cal/gm = 2268 J/gm

**Application**

(1) To convert  $m$  mass of ice at 0°C into steam at 100°C, amount of heat required is

$$80 m + 100 m + 540 m = 720 m \text{ cal}$$

(2) (a) Two objects having masses  $m_1, m_2$ , specific heat capacities  $c_1, c_2$  and temperatures  $t_1$  and  $t_2$  are mixed. If there is no change in state during mixing, then resulting temperature of mixture,

$$t_{mix} = \frac{m_1 c_1 t_1 + m_2 c_2 t_2}{m_1 c_1 + m_2 c_2}$$

(b) Specific heat of the mixture,  $c_{mix} = \frac{m_1 c_1 + m_2 c_2}{m_1 + m_2}$

(3)  $w$  gm of water at  $T$ °C is mixed with  $m$  gm of ice at 0°C.

(a)  $w = \frac{80m}{T} \Rightarrow$  Whole of ice melts. Final temperature = 0°C.

(b)  $w < \frac{80m}{T} \Rightarrow$  Whole of ice will not melt. Final temperature = 0°C

Amount of ice melted,  $m' = \frac{wT}{80}$

Amount of ice left =  $m - m'$

(c)  $w > \frac{80m}{T} \Rightarrow$  Whole of ice melts. Final temperature =  $\frac{wT - 80m}{w + m} > 0^\circ\text{C}$

## KINETIC THEORY OF GASES

Assumptions for ideal gas are

- (i) Kinetic theory of gases is applicable for large number of molecules.
- (ii) Intermolecular forces between two molecules is negligible.
- (iii) The force due to gravity on the molecules is neglected.
- (iv) The separation between the molecules is much larger as compared to their size.
- (v) The molecules are perfectly elastic and all collisions between the molecules and a wall are considered to be perfectly elastic.

All gases at high temperature and low pressure behave like an ideal gas.

### Pressure Exerted by the Gas

The pressure of the gas is due to continuous bombardment of the gas molecules against the walls of the container. According to kinetic theory, the pressure exerted by an ideal gas is given by

$$P = \frac{1}{3} \frac{M}{V} \bar{v}^2$$

$M$  = Mass of the enclosed gas

$V$  = Volume of the container

$\bar{v}^2$  = Mean square speed of molecules

Or 
$$P = \frac{1}{3} \frac{Nm}{V} \bar{v}^2$$

$N$  = Number of molecule

$m$  = Mass of the molecule

$\bar{v}^2$  = Mean square speed of molecules

Types of speed defined for a gas :

(i) Root mean square speed, 
$$\bar{v}_{rms} = \sqrt{\frac{3RT}{M_w}} = \frac{\sqrt{3P}}{\rho} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}}$$

(ii) 
$$\bar{v}_{Avg} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8P}{\rho}} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n}$$

(iii)  $v_{mp}$  = Most probable speed is defined the speed corresponding to which there are maximum number of molecules.

$$v_{mp} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2P}{\rho}}$$

$\rho$  = Density of gas  
 $M_w$  = Molecular weight  
 $R$  = Gas constant  
 $P$  = Pressure of gas

Order of magnitude :  $v_{rms} > v_{avg} > v_{mp}$

$$v_{rms} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$$

### Kinetic Interpretation of Temperature

$$\text{Translation Kinetic Energy} = \frac{3}{2} nRT$$

**THERMODYNAMICS**

**Internal Energy** : Every bulk system consists of a large number of molecules. Internal energy is simply the sum of the kinetic energies and potential energies of these molecules. It is a macroscopic variable.

$$\text{For gases } \Delta U = \frac{f}{2} nR\Delta T = \frac{f}{2} (P_2V_2 - P_1V_1)$$

**Law of equipartition of energy :**

According to law of equipartition of energy, the total kinetic energy of a Thermo dynamical system consisting of a large number of particles is equally distributed among its degree of freedom and hence

the average kinetic energy of a molecule associated with each degree of freedom is  $K.E. = \frac{1}{2} kT$  where  $k$  is Boltzmann constant and  $T$  is absolute temperature.

Gas	Degrees of freedom (f)	$\Delta U = \frac{f}{2} nR\Delta T$	$C_V = \frac{\Delta U}{n\Delta T}$	$C_P = C_V + R$	$\gamma = \frac{C_P}{C_V}$
Monoatomic	3 (Translational)	$\frac{3}{2} nR\Delta T$	$\frac{3}{2} R$	$\frac{5}{2} R$	$\frac{5}{3}$
Diatomic or Linear Polyatomic	3(Trans) + 2(Rot)	$\frac{5}{2} nR\Delta T$	$\frac{5}{2} R$	$\frac{7}{2} R$	$\frac{7}{5}$
Non-Linear Poly atomic	3 (Trans) + 3 (Rot)	$3nR\Delta T$	$3 R$	$4 R$	$\frac{4}{3}$

**Meanfree Path ( $\lambda$ )**

The meanfree path is the average distance covered by a molecule between two successive collision.

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n} \quad n = \text{No. of molecules per unit volume}$$

It is inversely proportional to density of gas.

In closed container if temperature is increased  $\lambda$  remain same.

**First Law of Thermodynamics** : It is law of conservation of energy.

Let  $Q$  heat is supplied to gas. It is used in two ways.

- (1) Increasing internal energy (i.e., increasing temperature).
- (2) Work done by the gas during expansion ( $W$ )

$$\Rightarrow Q = \Delta U + W$$

**Note** : (1)  $Q$  and  $W$  are path functions. They depend on the type of process.

(2)  $\Delta U$  is state function. It depends only on initial and final state of system.

$\Delta U = nC_V\Delta T$  (Always applicable, whatever may be the process. Here  $C_V$  is Molar heat capacity at constant volume)

**Thermodynamic Process****(1) Melting process :** (Change of state, solid to liquid)

$$Q = \Delta U + W$$

$$mL_f = \Delta U + 0 \quad [W = 0 \text{ as volume remains nearly constant}]$$

**(2) Boiling process :** (Change of state, liquid to gas)

$$mL_v = \Delta U + P[V_2 - V_1]$$

$V_2$  = volume of vapours

$V_1$  = volume of liquid

When 1 g of water vapourises isobarically at atmospheric pressure.  $\Delta U = 2091$  J,  $P = 1.01 \times 10^5$  Pa,  $V_1 = 1$  cm<sup>3</sup>,  $V_2 = 1671$  m<sup>3</sup>.

**(3) Isochoric process :**

$$dV = 0 \Rightarrow W = 0 \quad [dV = \text{change in volume}]$$

$$Q = nC_v\Delta T = \Delta U$$

$$\Rightarrow C_v = \frac{\Delta U}{n\Delta T}$$

**(4) Isobaric process :**

$$P = \text{constant}, dW = PdV$$

$$\Rightarrow W = P\Delta V = nR\Delta T$$

$$Q = nC_p\Delta T = \Delta U + W$$

$$nC_p\Delta T = nC_v\Delta T + nR\Delta T$$

$$\Rightarrow C_p = C_v + R$$

$$\boxed{\frac{\Delta U}{\left(\frac{f}{2}\right)} = \frac{W}{1} = \frac{Q}{\left(\frac{f+2}{2}\right)}} \quad \text{or} \quad \frac{\Delta U}{1} = \frac{W}{\gamma-1} = \frac{Q}{\gamma}$$

$$\text{Fraction of total heat converted to internal energy} = \frac{\Delta U}{Q} = \frac{1}{\gamma}$$

$$\text{Fraction of total heat converted to work is, } \frac{W}{Q} = \frac{\gamma-1}{\gamma}$$

**(5) Isothermal process :**

$$PV = K \Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0, C = \infty$$

To calculate the amount of work done by the gas in an isothermal process.

$$\text{as } PV = nRT \quad (\text{Constant})$$

$$\text{So } P = \frac{nRT}{V}$$

Work done

$$W = Q = nRT \log_e \frac{V_2}{V_1} = 2.303 nRT \log_{10} \frac{V_2}{V_1}$$

**(6) Adiabatic process :**

$$PV^\gamma = K \quad [\text{Equation of adiabatic process}]$$

$$\text{As } Q = 0, \quad nC\Delta T = 0 \text{ or } C = 0$$

$$\text{Also, } 0 = nC_V\Delta T + W \quad [\text{by first law of thermodynamics}]$$

$$\text{Now, } \boxed{W = -\Delta U}$$

**(7) Polytropic Process**

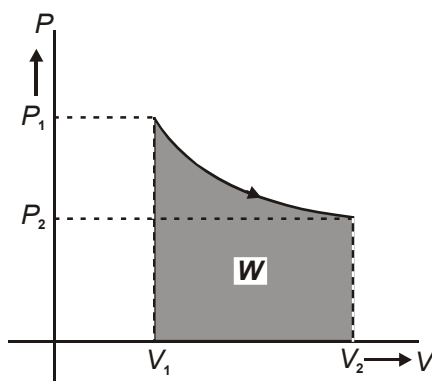
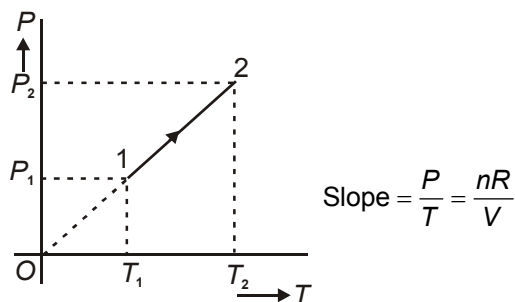
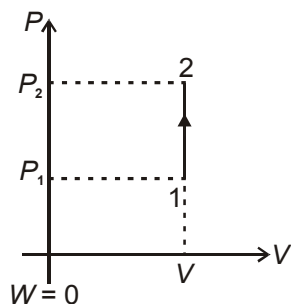
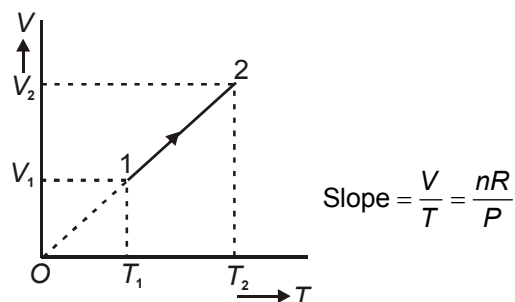
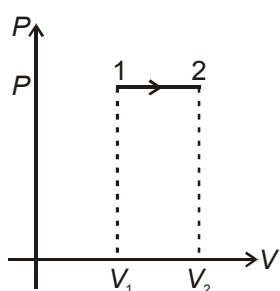
$$PV^n = \text{Constant}$$

$$W = \frac{nR\Delta T}{1-n}$$

$$C = C_V + \frac{R}{1-n}$$

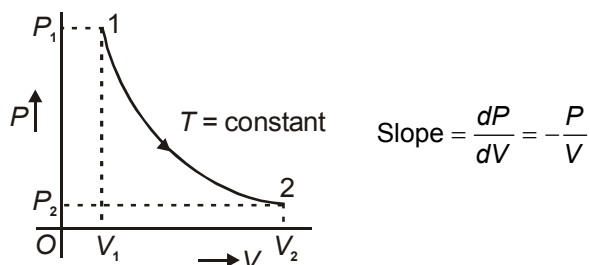
**Indicator Diagram:**

$P$ - $V$  graph of a process is called indicator diagram. Area under  $P$ - $V$  graph represents the work done in a process.

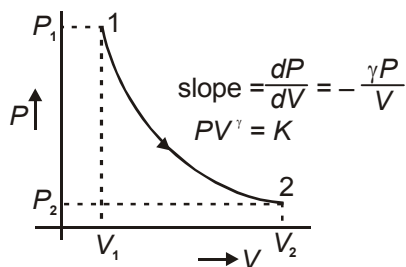
**Isochoric Process :****Isobaric :**

$$W = P(V_2 - V_1) = P\Delta V = nR\Delta T$$

**Isothermal process :**



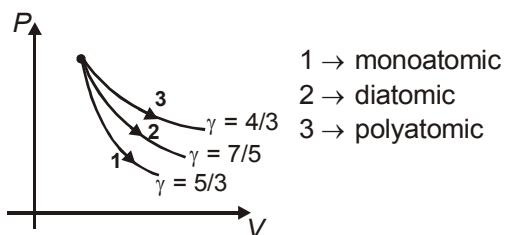
**Adiabatic process :**



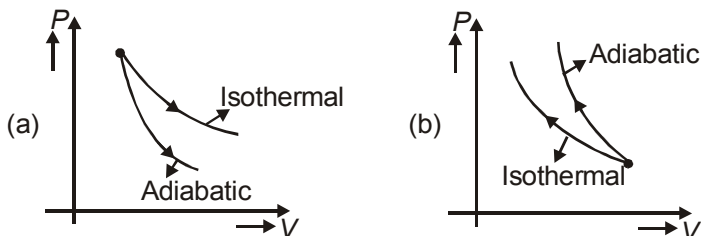
**Note :** Slope of adiabatic curve =  $\gamma$  (slope of isothermal curve)

**Applications**

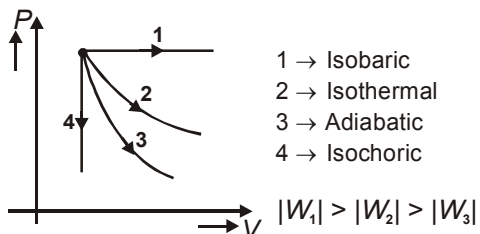
(1) P-V graph for different gases for adiabatic expansion



(2) P.V. graph for isothermal & adiabatic expansion & compression for a given gas

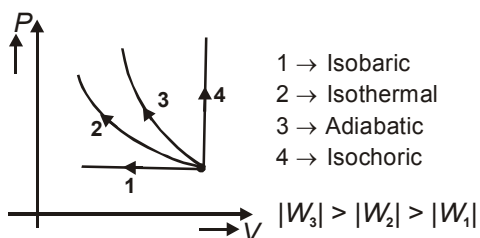


(3) **Expansion** of a gas under different processes





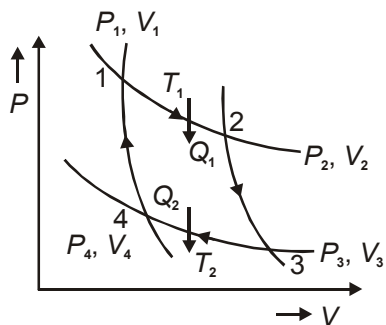
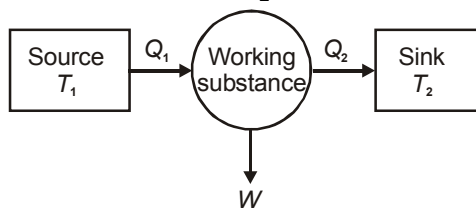
(4) Compression



**CARNOT ENGINE**

**Carnot Cycle :**

In a carnot engine the working substance (an ideal gas) draws some heat from the source per cycle (say  $Q_1$ ) performs some work  $W$  per cycle and rejects heat  $Q_2$  to the sink per cycle.



1 → 2	Isothermal Expansion	$\Delta U = 0$ $W_1 = Q_1 = nRT \ln \frac{V_2}{V_1}$ (positive)
2 → 3	Adiabatic Expansion	$Q = 0$ $W_2 = -\Delta U = \frac{nR\Delta T}{1-\gamma}$
3 → 4	Isothermal Compression	$\Delta U = 0$ $W_3 = Q_2 = nRT \ln \left( \frac{V_4}{V_3} \right)$ (negative)
4 → 1	Adiabatic Compression	$Q = 0$ $W_4 = -\Delta U = \frac{nR\Delta T}{1-\gamma}$

Heat supplied =  $Q_1$

Heat rejected =  $Q_2 \Rightarrow Q_1 - Q_2 = W$

$$\eta = \frac{W_{\text{total}}}{Q_{\text{supplied}}} \times 100 = \frac{Q_1 - Q_2}{Q_1} \times 100$$

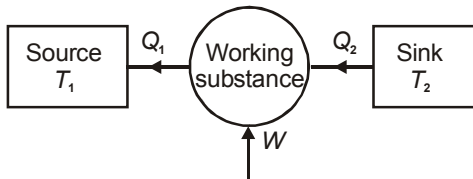
$$= \left( 1 - \frac{Q_2}{Q_1} \right) \times 100$$

$$= \left( 1 - \frac{T_2}{T_1} \right) \times 100 \quad (\text{for ideal engine})$$

$$Q_1 : Q_2 : W = T_1 : T_2 : T_1 - T_2 \text{ or } \frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{W}{T_1 - T_2}$$

**Carnot Theorem :** The efficiency of carnot engine is maximum (<100%) for given temperatures  $T_1$  and  $T_2$ . Any other engine working between temperature range  $T_1$  &  $T_2$  cannot have efficiency more than the carnot engine working between the same temperature range.

**Heat Pump :** In a heat pump  $W$  work is done on the working substance per cycle,  $Q_2$  heat is absorbed by the substance from lower temperature  $T_2$  per cycle and  $Q_1$  heat is supplied to higher temperature  $T_1$  ( $T_1 > T_2$ ) per cycle.



$$\beta = \frac{\text{Heat Supplied}}{W_{\text{total}}} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$$

$$\beta = \frac{T_1}{T_1 - T_2} \text{ (for ideal pump)}$$

**Relation between  $\eta$  and  $\beta$ ,**  $\eta = \frac{1}{\beta + 1}$  and  $\beta = \frac{1}{\eta} - 1$

**Refrigerator :** In a refrigerator,  $W$  work is done on the working substance,  $Q_2$  heat is absorbed from lower temperature  $T_2$  and  $Q_1$  heat is rejected to higher temperature  $T_1$ . ( $T_1 > T_2$ ).

$$\text{Coefficient of performance } \beta = \frac{\text{heat rejected}}{W_{\text{total}}} = \frac{Q_2}{Q_1 - Q_2}$$

$$\Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$

